Key Sector Analysis: A Case of the Transited Polish Economy

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The transition process from a centrally planned economy to a market economy started in Poland at the beginning of the 1990s. In this paper we try to answer the question in which direction has the structure of Polish economy changed, if indeed it has. By means of the key sector analysis applied to the Polish input-output tables that come from the period 1990–2000, we find that the structure of the Polish economy still remains characteristic of a centrally planned economy rather than a market economy. Although, in the last year of the period under study, the first improvement symptoms could be observed (the increased significance of services in the Polish economy) but there is still a lot of work to be done. An inefficient operation in the case of some sectors reaches a considerable level. This is reflected by the structure of the most important input-output coefficients, of which, the most important inputs are located on the diagonal of the sensitive matrix.

Key Words: input-output tables, transition, key sector analysis
JEL *Classification:* C67, P21

Introduction

In the early 1990s the centrally planned Polish economy started transforming into a market economy. Without any economic theory on how to carry out such a process, this task appeared to be very difficult. In the first stage of the transition, the economy in Poland suffered from the two opposing trends in the macroeconomic statistics. On the one hand, industrial output, wages and salaries dropped considerably and on the other, inflation and unemployment rose. Although, Balcerowicz's plan based on the three nominal anchors allowed inflation to be kept under control, the other macroeconomic statistics still remained below an advisable level. The first improvement symptoms could be observed in

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trade, and later also in services. Manufacturing, especially in the case of heavy industry, appeared to be extremely resistant to change.

The transition process did not avoid the basic principles used by the Central Statistical Office in Poland when preparing the input-output tables. For example, the Polish input-output table from 1990 was compiled according to the material product system (MPS), while the table for 1995 and later was made according to the system of national accounts (SNA). Therefore, the first task was to transform the input-output matrix from 1990 into the SNA system to assure the comparability of the matrices coming from different periods and the results of the computations based on these matrices.

Under this study, we decided to analyse the changes of the economic structure by means of methods based on the entropy theory (key sector analysis).

In 1967 Theil published his work on the entropy decomposition analysis, which provided a useful way of examining errors or changes in input structures. Theil (1967) suggested that a change could be decomposed into a set of additive components and he formulated a maximum entropy principle. On the basis of this principle, so-called key sector analysis can be performed. Key sector analysis provides empirical evidence regarding the economic structure of sectors within an economy (Chenery and Watanabe 1958; Hewings and Romanos 1981; Hewings 1982; Defourny and Thorbecke 1984; Białas and Gurgul 1998). The main aim of the key sector investigations is to find the sectors whose structure has the greatest impact on the rest of the economy. A key sector analysis of backward and forward linkages is related to the so-called multiplier product matrix (MPM). This matrix is based on the maximum entropy criterion. By means of this matrix, the probable future course of economic development can be identified. The interaction of the different firm strategies towards innovation explains the dynamics of the entries in the input-output matrix. Rasmussen (1956) introduced to economics the notions of backward and forward linkages. These two indices allow us to find the key sectors of an economy.

The Entropy Decompositions

A static IO model (Ćmiel and Gurgul 1996a; 1996b; 1997) is given by the equation

$$y = (\mathsf{I} - \mathsf{A})x, \tag{1}$$

where A denotes the IO matrix, $\boldsymbol{x}$ the output vector, and $\boldsymbol{y}$ the final demand vector.

Therefore,

$$\boldsymbol{x} = (\mathsf{I} - \mathsf{A})^{-1}\boldsymbol{y} = \mathsf{B}\boldsymbol{y}. \tag{2}$$

Define

$$B_{i\bullet} = \sum_{j=1}^{n} b_{ij}, \tag{3}$$

$$B_{\bullet j} = \sum_{i=1}^{n} b_{ij}, \quad \text{and} \tag{4}$$

$$V = \sum_{i,j=1}^{n} b_{ij}. \tag{5}$$

The input-output multiplier matrix (MPM) (Sonis and Hewings 1989) is given by the formula:

$$\mathsf{M} = \frac{1}{V}[B_{i\bullet}B_{\bullet j}] = \frac{1}{V}\begin{bmatrix} B_{1\bullet} \\ B_{2\bullet} \\ \vdots \\ B_{n\bullet} \end{bmatrix}[B_{\bullet 1}, B_{\bullet 2}, \ldots, B_{\bullet n}] = [m_{ij}]. \tag{6}$$

Thus, the structure of the MPM is essentially connected with the properties of the sectoral backward and forward linkages defined below.

Assuming that $B_{\bullet j_0}$ stands for the largest column multiplier and $B_{i_0\bullet}$ for the largest row multiplier, then the element located at (i_0, j_0) is given by the formula:

$$m_{i_0 j_0} = \frac{1}{V}B_{i_0\bullet}B_{\bullet j_0}, \tag{7}$$

which is called the largest cross. If this cross is excluded from M, then the second largest cross can be found. After the exclusion of row i_0 and j_0, the second largest cross $m_{i_0 j_0}$ can be found and so on. The number of crosses is equal to the rank of the matrix MPM. Following Rasmussen (1956), there are two types of indices of the Leontief inverse, which are called backward linkages

$$BL_j = \frac{\frac{1}{n}\sum_{i=1}^{n} B_{ij}}{\frac{1}{n^2}\sum_{i,j=1}^{n} B_{ij}} = \frac{nB_{\bullet j}}{V}, \tag{8}$$

and forward linkages

$$FL_i = \frac{\frac{1}{n}\sum_{j=1}^{n} B_{ij}}{\frac{1}{n^2}\sum_{i,j=1}^{n} B_{ij}} = \frac{nB_{i\bullet}}{V}. \tag{9}$$

A backward linkage greater than 1 ($BL_j > 1$) means that a unit change in final demand in sector j will create an above average increase in activity in the economy, and analogously if forward linkage is greater than 1 ($FL_i > 1$), it is taken for granted that a unit change in all sectors of the final demand will create an above average increase in sector i.

Definition 1. Sector k is called key sector if both indices are greater than 1.

Definition 2. Sector k is forward linkage oriented if FL is above 1 and BL is below 1.

Definition 3. Sector k is backward linkage oriented if FL is below 1 and BL is above 1.

Definition 4. Sector k is called weak oriented if both indices are less than 1.

Assuming $\mathsf{B} = (\mathsf{I} - \mathsf{A})^{-1} = [b_{ij}]$ to be Leontief inverse, also known as the matrix of total inputs. For each location (i_0, j_0) we define a matrix $\mathsf{F}(i_0, j_0)$ (Sonis and Hewings 1989; 1992) of the form:

$$\mathsf{F}(i_0, j_0) = \begin{bmatrix} B_{1i_0} \\ B_{2i_0} \\ \vdots \\ B_{ni_0} \end{bmatrix} [B_{j_01}, B_{j_02}, \ldots, B_{j_0n}] = [b_{ii_0} b_{j_0j}]. \tag{10}$$

This matrix is referred to as the *first order field of influence of change.* The economic interpretation of this matrix is related to the Sherman-Morrison (1950) formula. By using this formula, changes of entries in matrix B can be investigated. If the change e is located in position (i_0, j_0) in matrix A, following the above mentioned Sherman-Morrison formula, we have

$$b_{ij}^{e(i_0,j_0)} = b_{ij} + \frac{b_{ii_0} b_{j_0j} e}{1 - b_{j_0i_0} e}, \tag{11}$$

or in matrix form

$$\mathsf{B}^{\varepsilon(i_0,j_0)} - \mathsf{B} = \frac{e}{1 - b_{j_0i_0}} \mathsf{F}(i_0, j_0). \tag{12}$$

The sum of all components of the matrix $\mathsf{F}(i_0, j_0)$ is given below:

$$S(\mathsf{F}(i_0, j_0)) = \sum_{i,j} b_{ii_0} b_{j_0j} = B_{\bullet i_0} B_{j_0 \bullet} \quad \text{and} \tag{13}$$

$$\mathsf{M} = \frac{1}{V} [S(\mathsf{F}(j, i))]. \tag{14}$$

These formulas allow us to investigate the importance of direct inputs, which means the impact of the coefficients of A on $\mathsf{B} = (\mathsf{I} - \mathsf{A})^{-1}$.

Below we demonstrate that MPM has the property of maximum entropy (Shannon and Weaver 1964; Theil 1967; Kullback 1970).

Let $\mathsf{Q} = [Q_{ij}]$ be a positive matrix and

$$\sum_j Q_{ij} = B_{i\bullet}, \quad \sum_i Q_{ij} = B_{\bullet j} \quad \text{and} \quad \sum_{ij} Q_{ij} = V. \tag{15}$$

Consider the probability spaces $(\mathcal{X}, \mathcal{F}, P_i)$ $i = 1, 2$, that is a basic set of elements $x \in \mathcal{X}$ and a collection $\mathcal{F}$ of all possible events (sets) made up of elements of the sample space $\mathcal{X}$ for which a probability measure P_i, $i = 1, 2$ has been defined. Assuming that the probability measures P_1 and P_2 are absolutely continuous with respect to one another, then there exists a probability measure μ (for example $\mu = (P_1 + P_2)/2$) and functions $f_i(x)$, $i = 1, 2$ called the generalized probability densities (Radon-Nikodym derivatives), unique up to sets of probability zero in μ, measurable in μ, $0 < f_i(x) < \infty$ almost everywhere in μ such that, for all $A \in \mathcal{F}$, $P_i(A) = \int_A f_i(x) d\mu(x)$, $i = 1, 2$.

Applying the Taylor expansion

$$\log x = x - 1 - \frac{1}{2c^2}(x-1)^2, c \in (\min\{1, x\}, \max\{1, x\}),$$

one can see that

$$\begin{aligned}
&\int_{\mathsf{X}} f_1(x) \log \frac{f_2(x)}{f_1(x)} d\mu(x) \\
&\quad = \int_{\mathsf{X}} f_1(x) \left\{ \frac{f_2(x)}{f_1(x)} - 1 - \frac{1}{2c^2} \left(\frac{f_2(x)}{f_1(x)} - 1 \right)^2 \right\} d\mu(x) \\
&\quad = \int_{\mathsf{X}} (f_2(x) - f_1(x)) d\mu(x) - \frac{1}{2c^2} \int_{\mathsf{X}} f_1(x) \left(\frac{f_2(x)}{f_1(x)} - 1 \right)^2 d\mu(x) \\
&\quad = \int_{\mathsf{X}} f_2(x) d\mu(x) - \int_{\mathsf{X}} (f_1(x) d\mu(x) \\
&\qquad - \frac{1}{2c^2} \int_{\mathsf{X}} f_1(x) \left(\frac{f_2(x)}{f_1(x)} - 1 \right)^2 d\mu(x) \\
&\quad = 1 - 1 - \frac{1}{2c^2} \int_{\mathsf{X}} f_1(x) \left(\frac{f_2(x)}{f_1(x)} - 1 \right)^2 d\mu(x) \le 0.
\end{aligned}$$

The inequality

$$\int_{\mathsf{X}} f_1(x) \log \frac{f_2(x)}{f_1(x)} d\mu(x) \le 0$$

is known as the basic information inequality.

Applying this inequality to the two dimensional distributions with the density function $f_{XY}(x,y)$ and the product of one dimensional distribution $f_X(x)f_Y(y)$ we have

$$\int_{\mathsf{X}} f_{XY}(x,y)\log\frac{f_X(x)f_{Y(y)}}{f_{XY}(x,y)}d\mu(x,y)\le 0,$$

and as a consequence

$$\int_{\mathsf{X}} f_{XY}(x,y)\log f_{XY}(x,y)d\mu(x,y)$$
$$\le -\int_{\mathsf{X}} f_X(x)\log f_X(x)d\mu(x)-\int_{\mathsf{X}} f_Y(y)\log f_Y(y)d\mu(y).$$

The above inequality can be written in the form

$$H(X,Y)\le H(X)+H(Y),$$

where

$$H(X)=-\int_{\mathsf{X}} f_X(x)\log f_X(x)d\mu(x)$$

is called the entropy of random variable X (or its distribution).

Similarly

$$H(X,Y)=-\int_{\mathsf{X}} f_{XY}(x,y)\log f_{XY}(x,y)d\mu(x,y)$$

is called the entropy of a two dimensional random variable (X,Y) (or its distribution).

For the discrete two dimensional distribution we have (in a special case) the inequality

$$\sum_{i,j} p_{i,j}\log\frac{p_{i\bullet}p_{\bullet j}}{p_{ij}}\le 0.$$

Hence

$$-\sum_{i,j} p_{i,j}\log p_{i,j}\le -\sum_i p_{i\bullet}\log p_{1\bullet}-\sum_j p_{\bullet j}\log p_{\bullet j}. \tag{16}$$

Applying this result to the probabilistic distribution $\mathsf{Q}(p_{ij}=Q_{ij}/V)$ and the product M of its marginals ($p_{i\bullet}=B_{i\bullet}/V$ and $p_{\bullet j}=B_{\bullet j}/V$) and taking into account

$$H(\mathsf{Q})=-\sum_{i,j} p(i,j)\ln p(i,j)=-\sum_{i,j}\frac{Q_{i,j}}{V}\ln\frac{Q_{i,j}}{V} \tag{17}$$

we have

$$H(\mathsf{Q})=-\sum_{i,j}\frac{Q_{i,j}}{V}\ln\frac{Q_{i,j}}{V}\le-\sum_{i,j}\frac{B_{i\bullet}B_{\bullet j}}{V^2}\left(\ln\frac{B_{i\bullet}}{V}+\ln\frac{B_{\bullet j}}{V}\right)$$

$$= -\sum_{i,j} \frac{B_{i\bullet}B_{\bullet j}}{V^2} \ln \frac{B_{i\bullet}B_{\bullet j}}{V^2} = H(\mathsf{M}). \tag{18}$$

The multiplier product matrix M depends on the column and row multipliers. Therefore, MPM does not take into account the interactions of each sector with other sectors. From (18) it follows that MPM has the property of maximal entropy in the class of all matrices with fixed marginals. For the case where simultaneous changes occur in two places (i_0, j_0) and (i_1, j_1) in a direct inputs matrix, a formula similar to (11) can be derived (see Hewings and Romanos 1981).

Notice that the sum of all the elements of M is equal to the sum of all the elements of B.

Matrix M represents the maximum entropy tendency. Thus, matrix M may be considered to represent the most homogenous distribution of the components of the column and row multipliers of the Leontief inverse B (which represents total inputs). From the economic point of view, the MPM matrix stands for the equalisation tendency of total inputs in an economy (i. e. in all industries the same – in monetary approach – output needs approximately the same input).

Define

$$\mathsf{D} = \mathrm{diag}(\mathsf{B} - \mathsf{M}) \quad \text{(diagonal)},$$
$$\mathsf{R} = \mathsf{B} - \mathsf{M} - \mathsf{D},$$
$$\mathsf{S} = 2^{-1}(\mathsf{R} + \mathsf{R}^T) \quad \text{(symmetric with null diagonal)},$$
$$\mathsf{S}_a = 2^{-1}(\mathsf{R} - \mathsf{R}^T) \quad \text{(asymmetric with null diagonal)}.$$

Therefore,

$$\mathsf{R} = 2^{-1}(\mathsf{R} + \mathsf{R}^T) + 2^{-1}(\mathsf{R} - \mathsf{R}^T) = \mathsf{S} + \mathsf{S}_a.$$

Thus

$$\mathsf{B} = \mathsf{M} + \mathsf{D} + \mathsf{S} + \mathsf{S}_a,$$

where M represents the maximum entropy tendency and the diagonal matrix D stands for the so-called additional sectoral scale effects. The symmetric matrix S and asymmetric matrix S_a represent the symmetric and asymmetric tendency.

Application to the Polish Input-Output Tables from the Period 1990–2000

The statistical data used here come from the Polish Statistical Yearbooks (published by The Central Statistical Office) covering the period 1990–2000. Since input-output tables are published on a basis of a five-year-period in Poland, we focus our attention on the three years in the period

TABLE 1 Key Sector analysis of the Polish economy over the period 1990–2000 (aggregation 6×6)

Sector	1990		1995		2000	
	FL	BL	FL	BL	FL	BL
1. Manufacturing	2.295	1.070	2.646	1.258	1.556	1.038
2. Construction	0.536	0.988	0.526	1.087	0.731	1.081
3. Agriculture and forestry	0.852	1.122	0.854	1.045	0.854	1.167
4. Transportation and communication	0.736	1.057	0.691	1.022	0.832	0.980
5. Trade	0.687	0.894	0.536	0.780	0.979	0.853
6. Service	0.894	0.869	0.747	0.808	1.047	0.881

of interest, namely: 1990, 1995, and 2000. We began by using an aggregation 6×6. Using a notion of backward and forward linkages, the taxonomy of the Polish economy is carried out. This taxonomy characterises changes in the economic structure over the period under consideration (Sonis and Hewings 1981; Durand and Markle 1984). The results of key sectors analysis are summarised in table 1 and figures 1, 2, and 3.

Over the whole analysed period, the key sector was manufacturing. This indicates that a unit change in final demand in this sector will create an above average increase in activity in the economy, and unit change in all sectors of the final demand will create an above average increase of output in this sector. It is worth noting that both the backward and forward linkages of this sector tend to increase only in the first half of the analysed period, while in 2000 their level is even below the one observed in 1990. This means that the influence of the changes in the final demand of this sector on the whole economy was initially becoming stronger with time, but then the trend was reversed. It should also be emphasised that there is no forward linkage oriented sector at this level of aggregation except for services in 2000. The backward oriented sectors were agriculture and forestry as well as transportation and communication, but in the case of the second one, only over the first half of the analysed period. Also, construction could be considered as a backward linkage oriented sector since 1995, when its *BL* reached the greatest value. Apart from the key sectors, it is also interesting which of the selected sectors can be categorised as a weakly oriented sector. It results from our computations that trade alone is a weakly oriented sector over the whole period under consideration.

FIGURE 1 Visualisation of the results of key sector analysis for 1990

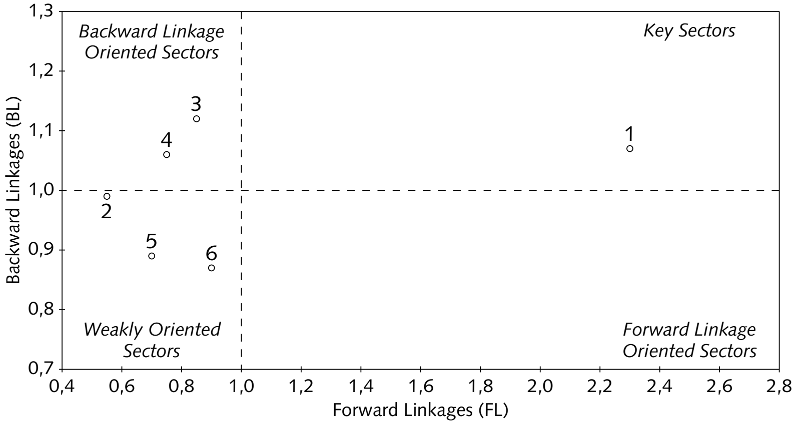


FIGURE 2 Visualisation of the results of key sector analysis for 1995

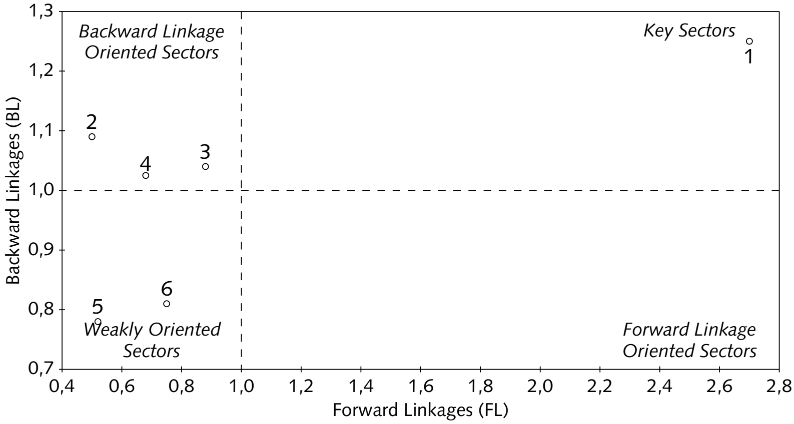
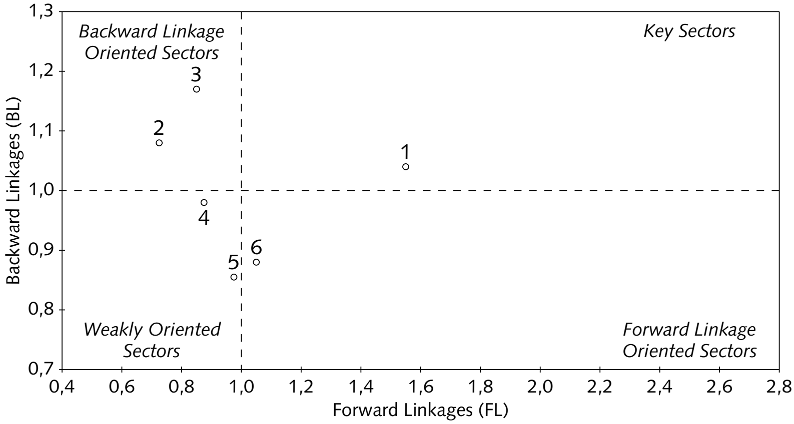


FIGURE 3 Visualisation of the results of key sector analysis for 2000



The investigations of the most important coefficients were performed by means of the Sherman-Morrison formula defined by (11), which gives the change in the entries of the Leontief inverse, caused by a change in one component of the direct coefficients matrix A. For any fixed position (i_0, j_0) we perturbed the corresponding element of A by replacing the element a_{i_0,j_0} by $a_{i_0,j_0}(1+\varepsilon)$. Let us denote the direct coefficients matrix perturbed in this way by $\mathsf{A}^{\varepsilon(i_0,j_0)}$. The number can be interpreted as the relative perturbation of a_{i_0,j_0}. Then the inverse error matrix given by

$$\mathsf{B}^{\varepsilon(i_0,j_0)} = \left[b_{ij}^{\varepsilon(i_0,j_0)}\right] = (\mathsf{I}-\mathsf{A}^{\varepsilon(i_0,j_0)})^{-1} - (\mathsf{I}-\mathsf{A})^{-1},$$

the relative inverse error matrix

$$\mathsf{RB}^{\varepsilon(i_0,j_0)} = \left[\frac{b_{ij}^{\varepsilon(i_0,j_0)}}{b_{ij}}\right], \quad \text{and its norm}$$

$$\|\mathsf{RB}^{\varepsilon(i_0,j_0)}\| = \sum_{i,j=1}^{n} \left|\frac{b_{i,j}^{\varepsilon(i_0,j_0)}}{b_{i,j}}\right|$$

were computed.

In the same way we computed the above elements for all possible positions (i_0, j_0), so we finally obtained the sensitivity matrix

$$\mathsf{SM} = \left[\|\mathsf{RB}^{\varepsilon(i_0,j_0)}\|\right].$$

Then we assigned ranks in descending order for the elements of this matrix. Rank 1 in position (i_0, j_0) indicates that a_{i_0,j_0} is the most sensitive (according to the inverse) element of matrix A, i. e. the relative change of this element has the greatest influence on $\mathsf{B} = (\mathsf{I}-\mathsf{A})^{-1}$.

The analysis was done, in the first step, for the years 1990, 1995 and 2000 in an aggregation 6×6. The results are presented in table 2.

One will find that the most important input-output coefficients are manufacturing products used as input by manufacturing itself, agriculture and forestry products used by agriculture and forestry itself or by manufacturing. Among the most important coefficients in the analysed periods are also the coefficients of manufacturing products used by agriculture and forestry or by transportation and communication or by services, and service products used by manufacturing, and products of trading used by manufacturing. The input coefficients whose changes are less important for the economy under study are also interesting. The least important coefficients in the considered periods were those of construction products used by construction itself or trade, agriculture and

TABLE 2 Inverse important coefficients over the period 1990–2000 (aggregation 6 × 6)

	1990						1995						2000					
	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.
1.	**1**	**7**	**8**	**5**	11	**4**	**1**	**7**	4	6	11	**10**	**1**	**5**	**7**	**8**	17	**4**
2.	15	34	24	26	23	18	13	16	30	27	33	15	20	14	33	28	26	**10**
3.	**2**	33	**3**	36	30	35	**2**	35	**3**	36	26	34	**2**	36	**3**	35	30	34
4.	**10**	21	31	14	12	25	**8**	24	29	**9**	21	19	15	29	32	16	13	22
5.	**6**	29	20	19	32	27	12	25	31	20	32	18	**6**	23	19	21	25	24
6.	**9**	28	22	16	17	13	**5**	23	28	17	22	14	18	27	31	12	11	**9**

Note that inverse important coefficients are numbered according to importance.
The ten top inverse important coefficients are in bold.

forestry products used by construction or by transportation and communication or by services.

The authors also provided the results of the decomposition of the Leontief inverse into the M (called MPM), diagonal, symmetric and asymmetric matrices in the years under consideration (see tables 3 and 4).

The additional effect greater than one can be observed in the case of such sectors as: agriculture and forestry (1990, 1995, and 2000), manufacturing (1995) and also transportation and communication (1995). The smallest effect can be identified in the case of the trade sector. From matrix S it follows that the largest (by absolute value) bilateral balances occur in the pair of sectors: agriculture and forestry and transportation and communication (1990), manufacturing and services (1995) as well as agriculture and forestry and services (2000). From matrix S_a it follows that the largest (by absolute value) bilateral imbalances were in the pair of sectors construction and manufacturing (1990–2000).

The authors also performed the computations for two different aggregations: 10 × 10 and 24 × 24. Unfortunately, since 2000 the Central Statistical Office has published the Polish input-output tables on the basis of different schema that makes it impossible to prepare an aggregation 10 × 10 and 24 × 24. Therefore, we have used an input-output matrix updating technique based on the sum of squared differences. More details can be found in Jackson and Murray (2004). The selected results are summarized below (see tables 5, 6, and 7).

One will notice that the superscripts of top inverse important coefficients (obtained from 100 and 576 coefficients) are strongly related to the

TABLE 3 Matrix M over the period 1990-2000 (aggregation 6×6)

	1990						1995						2000					
	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.
1.	.873	.204	.324	.280	.262	.340	1.395	.277	.450	.364	.283	.394	.495	.233	.272	.265	.311	.333
2.	.806	.188	.299	.259	.241	.314	1.206	.240	.389	.315	.244	.340	.515	.242	.283	.275	.324	.346
3.	.916	.214	.340	.294	.274	.357	1.160	.231	.374	.303	.235	.327	.556	.261	.305	.297	.350	.374
4.	.863	.202	.320	.277	.258	.336	1.134	.225	.366	.296	.230	.320	.467	.219	.256	.250	.294	.314
5.	.730	.170	.271	.234	.219	.284	.865	.172	.279	.226	.175	.244	.406	.191	.223	.217	.256	.273
6.	.709	.166	.263	.228	.212	.276	.896	.178	.289	.234	.182	.253	.420	.197	.230	.225	.264	.282

TABLE 4 Matrix $[\mathsf{S}_a \backslash \mathsf{D} \backslash \mathsf{S}]$ over the period 1990–2000 (aggregation 6×6)

	1990						1995						2000					
	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.	1.	2.	3.	4.	5.	6.
1.	**.952**	–.143	–.226	–.171	–.211	–.199	**1.133**	–.161	–.210	–.243	–.241	–.278	**.915**	–.144	–.173	–.207	–.190	–.200
2.	–.644	**.838**	–.210	–.167	–.154	–.163	–1.009	**.926**	–.237	–.202	–.166	–.159	–.340	**.949**	–.246	–.200	–.175	–.184
3.	–.546	.062	**1.096**	–.248	–.195	–.217	–.684	.128	**1.107**	–.252	–.183	–.224	–.284	.017	**1.071**	–.240	–.167	–.245
4.	–.617	.069	–.014	**.890**	–.134	–.169	–.778	.081	–.047	**1.011**	–.156	–.157	–.182	.048	.037	**.935**	–.155	–.132
5.	–.431	.062	.014	.000	**.845**	–.150	–.564	.061	–.057	–.002	**.881**	–.135	–.053	.126	.159	.044	**.843**	–.156
6.	–.377	.128	.094	.121	.086	**.899**	–.456	.131	.029	.092	.050	**.953**	–.092	.111	.110	.130	.044	**.917**

Note that the diagonal elements of $[\mathsf{S}_a \backslash \mathsf{D} \backslash \mathsf{S}]$ are equal to the diagonal element of D. The lower triangular elements of $[\mathsf{S}_a \backslash \mathsf{D} \backslash \mathsf{S}]$ are equal to the lower triangular elements of S_a. Upper triangular elements of $[\mathsf{S}_a \backslash \mathsf{D} \backslash \mathsf{S}]$ are equal to upper triangular elements of S.

TABLE 5 Key sector analysis of the Polish economy over the period 1990–2000 (aggregation 10×10)

Sector	1990		1995		2000	
	FL	BL	FL	BL	FL	BL
1. Group of industries of fuels and energy	1.372	1.061	1.420	1.027	1.073	0.893
2. Group of industries of raw material	1.711	1.048	1.922	1.170	1.050	0.991
3. Electromachine	0.963	0.994	1.004	1.169	0.753	0.958
4. Food	0.656	1.073	0.711	1.239	0.820	1.292
5. Group of light industries	0.947	1.029	1.140	1.093	0.887	0.979
6. Construction	0.561	0.960	0.545	0.984	0.797	1.054
7. Agriculture and forestry	0.968	1.091	0.911	0.955	1.026	1.167
8. Transportation and communication	0.860	1.027	0.812	0.920	0.989	0.956
9. Trade	0.839	0.871	0.585	0.711	1.299	0.845
10. Services	1.122	0.846	0.949	0.731	1.306	0.865

key sectors numbers. Therefore our conjecture is that the parameters of location of inverse important coefficients determine approximately the same key sectors as indicators BL_j and FL_i.

We also find that the most important input coefficients come from the main diagonal of the input-output matrix (large *intraindustry* flows). This means that this structure of the most important inputs was still typical for centrally planned economies (domination of raw materials and fuels). Large coefficients $a_{i,i}$ can be a signal of the inefficiency of economy (*intraindustry* flows dominate *interindustries* flows – i. e. in some branches their production is mainly devoted to their input, therefore, some branches producing final goods can experience a shortage of intermediate goods needed for their own inputs). In highly developed market economies these coefficients are lower. In the considered periods we also find some important input-output coefficients connected with *interindustries* flows, for example, agriculture and forestry products used as input by the food industry, food products used as input by trade, metallurgical products used as input by the electromachine industry. In these three cases, the importance of these coefficients is somewhat natural.

Other conclusions (analogous to the case of aggregation 6×6) are left to the reader.

TABLE 6 Key sector analysis of the Polish economy over the period 1990–2000 (aggregation 24 × 24)

Sector	1990		1995		2000	
	FL	BL	FL	BL	FL	BL
1. Products of the coal and fuel industry	1.826	1.116	2.067	1.014	1.261	0.797
2. Energy, gas, hot water	1.273	1.031	1.223	1.049	1.212	1.042
3. Metal ores, products from metallurgic irons and non-ferrous industry, recycling of metals	1.921	1.107	2.004	1.409	1.288	1.254
4. Products of metal industry	0.863	0.992	0.880	1.113	0.876	1.061
5. Machinery and device	1.017	0.972	1.031	1.123	0.761	1.053
6. Products of precise industry	0.554	0.921	0.797	1.199	0.716	1.023
7. Products of the transport industry and the transportation trade	0.782	1.044	0.952	0.975	0.975	1.008
8. Products of electrotechnical industry	0.833	1.063	0.624	1.148	0.599	0.880
9. Chemicals and chemical products and products manufactured with other non-metal materials	1.765	1.072	2.703	1.104	1.227	1.002
10. Products of the wood industry but not including furniture	0.944	1.056	0.700	1.089	0.813	1.100
11. Products of the paper and printing industry, data carriers, remaining products and material services	1.109	1.017	1.279	1.168	0.921	0.986

Continued on the next page

Conclusions

The main aim of this paper is to carry out a taxonomy of the Polish economy in transition. A further interest is to identify the most important input-output coefficients and also answer the question as to whether or not the structure of the Polish economy is still characteristic for a centrally planned economies.

Applying the methods based on the entropy theory, we identified the sectors which can be considered as a key, and examined the additional

TABLE 6 *(continued)*

Sector	1990		1995		2000	
	FL	BL	FL	BL	FL	BL
12. Textiles	0.821	1.066	0.862	1.173	0.656	0.889
13. Clothes and products manufactured from fur, skin or products manufactured with skin	0.592	1.001	0.561	1.000	0.587	0.892
14. Food	0.879	1.094	0.819	1.248	0.962	1.343
15. Production and services of construction	0.717	0.980	0.733	1.000	1.125	1.102
16. Agriculture, hunting, forestry and fishing	1.251	1.112	1.074	0.956	1.165	1.211
17. Transportation	1.264	1.085	0.975	0.958	1.130	0.976
18. Communication	0.567	0.826	0.581	0.721	0.905	1.048
19. Trade	1.368	0.890	0.544	0.692	1.859	0.870
20. Municipal services, water and its distribution	0.604	1.145	0.467	0.846	0.760	0.937
21. Housing services	0.502	0.924	0.857	0.810	0.747	0.997
22. Education, medical services, social services	0.604	0.823	0.450	0.688	0.647	0.742
23. Services for people (hotels, restaurants, tourism, financial agency, leasing of machines and services)	0.803	0.916	1.447	0.841	2.244	1.021
24. Government administration, organizations	1.139	0.745	0.369	0.675	0.564	0.766

scale effects as well as the symmetric and asymmetric tendency in the economy. The striking empirical finding based on the most important input-output coefficient analysis is that the greatest importance is associated with the input coefficients which come from the diagonals of input-output matrices. It can be concluded that the structure of most important inputs was still typical for centrally planned economies even in 2000. In addition, large input coefficients $a_{i,i}$ may be a signal of an inefficiency of the Polish economy over the period under consideration.

It is worth noting that there exist also positive tendencies in the Polish economy. For example, the increased importance of services. In our opinion, this trend will continue also in the future.

TABLE 7 Ten top inverse important coefficients

Aggregation	Year	Coefficients
10×10	1990	$a_{2,2}$; $a_{1,1}$; $a_{7,7}$; $a_{5,5}$; $a_{7,4}$; $a_{4,9}$; $a_{2,3}$; $a_{1,2}$; $a_{3,3}$; $a_{9,2}$
	1995	$a_{2,2}$; $a_{1,1}$; $a_{5,5}$; $a_{3,3}$; $a_{4,4}$; $a_{7,7}$; $a_{2,3}$; $a_{7,4}$; $a_{8,8}$; $a_{10,2}$
	2000	$a_{1,1}$; $a_{3,1}$; $a_{3,3}$; $a_{1,6}$; $a_{1,2}$; $a_{5,1}$; $a_{1,3}$; $a_{1,4}$; $a_{6,6}$; $a_{2,6}$
24×24	1990	$a_{3,3}$; $a_{1,1}$; $a_{9,9}$; $a_{16,16}$; $a_{11,11}$; $a_{16,14}$; $a_{1,2}$; $a_{12,12}$; $a_{14,19}$; $a_{16,10}$
	1995	$a_{3,3}$; $a_{9,9}$; $a_{11,11}$; $a_{1,1}$; $a_{1,2}$; $a_{12,12}$; $a_{14,14}$; $a_{16,16}$; $a_{16,14}$; $a_{5,5}$
	2000	$a_{3,3}$; $a_{23,23}$; $a_{16,16}$; $a_{16,14}$; $a_{14,14}$; $a_{1,2}$; $a_{3,4}$; $a_{7,17}$; $a_{13,18}$; $a_{9,2}$

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References

Białas, S., and H. Gurgul. 1998. Hypothesis about the second eigenvalue of the Leontief matrix. *Economic Systems Research* 10:285–9.

Chenery, H. B., and T. Watanabe. 1958. International comparisons of the structure of production. *Econometrica* 26 (4): 487–521.

Ćmiel, A., and H. Gurgul. 1996a. Dynamic input-output models with stochastic time lags. *International Journal of Systems Science* 27:857–61.

Ćmiel, A., and H. Gurgul. 1996b. Dynamic input-output models with stochastic matrices and time lags. *Economic Systems Research* 8:133–43.

Ćmiel, A., and H. Gurgul. 1997. Stochastic backward-lag-type Leontief model. *The Central European Journal for Operations Research and Economics* 5:34–45.

Defourny, J., and E. Thorbecke. 1984. Structural path analysis and multiplier decomposition within a social accounting framework. *Economic Journal* 94:111–36.

Durand, R., and T. Markle. 1984. Diversity analysis of structural changes. *Economic Systems Research* 6 (3): 277–97.

Hewings, G. J. D. 1982. The empirical identification of key-sectors in an economy: A regional perspective. *The Developing Economies* 20:173–95.

Hewings, G. J. D., and M. C. Romanos. 1981. Simulating less developed regional economies under conditions of limited information. *Geographical Analysis* 13 (4): 373–90.

Jackson, R. W., and A. T. Murray. 2004. Alternative input-output matrix updating formulations. *Economic Systems Research* 16:135–48.

Kullback, S. 1970. *Infomation theory and statistics.* New York: Oxford University Press.

Rassmusen, P. 1956. Studies in inter-sectoral relations. Copenhagen: Einar Harks.

Shannon, C. E., and W. Weaver. 1964. The mathematical theory of communications. Urbana, IL: University of Illinois Press.

Sherman, J., and W. J. Morrison. 1950. Adjustment of an inverse matrix corresponding to a change in one element of a given matrix. *Annals of Mathematical Statistics* 21:124–7.

Sonis, M., and G. J. D. Hewings. 1992. Coefficients change in input-output model: theory and applications. *Economic Systems Research* 4:143–57.

Sonis, M., and G. J. D. Hewings. 1989. Errors and sensitivity in input-output analysis: new approach. In *Frontiers in input-output analysis,* eds. R. Miller, K. Polenske, and A. Rose. New York: Oxford University Press.

Theil, H. 1967. Economics and information theory. New York: Elsevier.